ECE 536 – Spring 2022

Homework #8 – Solutions

Problem 1)

(a) In the Fabry-Pérot cavity we have

$$\Delta\lambda = \frac{\lambda^2}{2n_g L}$$

The spacing between the two maxima is

$$\Delta\lambda = 810.65 - 809.825 = 0.825\,\text{nm}$$

Considering $\lambda = 809.825\,\text{nm}$, we obtain the group index

$$n_g = \frac{\lambda^2}{2\Delta\lambda L} \approx 3.9746$$

(b) From the measurement

$$\frac{I_{\max}}{I_{\min}} = \frac{2.77}{0.51} \approx 5.4314$$

so the parameter *A* is

$$A = \frac{\sqrt{\frac{I_{\max}}{I_{\min}}}-1}{\sqrt{\frac{I_{\max}}{I_{\min}}}+1} = \sqrt{R_1 R_2}\, e^{G_n L} \approx 0.3995$$

from which we solve for the modal gain

$$G_n = \frac{1}{L}\ln\frac{\sqrt{\frac{I_{\max}}{I_{\min}}}-1}{\sqrt{\frac{I_{\max}}{I_{\min}}}+1} + \frac{1}{2L}\ln\frac{1}{R_1 R_2}$$

$$= \frac{1}{100\times 10^{-4}}\ln\left(0.3995\right) + \frac{0.5}{100\times 10^{-4}}\ln\left(\frac{1}{0.3^2}\right) = 83.1496\ \text{cm}^{-1}$$

Problem 2)

(A) LOSS AND THRESHOLD The loss spectrum is shown in Fig. 2.1. The lasing wavelength should be the cavity mode with the lowest loss. For this case, we can reason that it will be right around 980nm, where the reflectivity band has the highest value (meaning the lowest mirror l oss). To estimate the threshold gain, we take the reflectivity to be 0.9851, 0.9325, and 0.3333 for R_{1a}, R_{1b} and R2 respectively. The gain numbers are shown in the solution for part (b). [Remember that threshold material gain requires normalizing by the confinement factor since part of the mode does not overlap with the gain.]

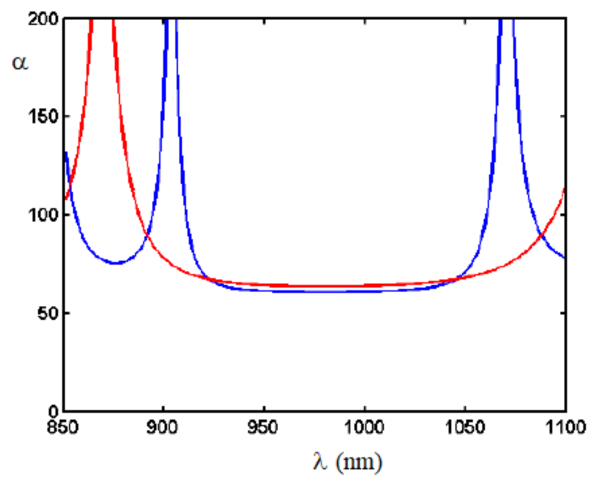

Figure 2.1: Loss spectrum for lasers in Problem 2.

(B) EMPIRICAL GAIN MODEL The empirical model in Coldren depends on both the transparency carrier density and another fitting parameter (N_s) as well as a gain coefficient g_0. It is given by

$$g = g_0 \ln \frac{N + N_s}{N_{tr} + N_s} \tag{2.1}$$

A plot of Coldren's model is shown in Fig. 2.2 along with the threshold gain values of the two lasers. The higher reflectivity laser (blue) has a threshold of 1896cm^{-1} and the lower reflectivity (red) is roughly 1982cm^{-1}. This gives threshold carrier densities of 4.4×10^{18} and 4.61×10^{18}cm^{-3} respectively.

(C) THRESHOLD CURRENT Coldren gives another empirical model to relate gain to injection current density that depends on the same parameters as above, written instead for current density. The relation is given as

$$J = (J_{tr} + J_s)\left[\frac{N + N_s}{N_{tr} + N_s}\right] e^{g_{0N}/g_{0J}} - J_s \tag{2.2}$$

This gives us a threshold current density of 597A/cm^2 and 628A/cm^2, corresponding to 2.99mA and 3.14mA respectively.

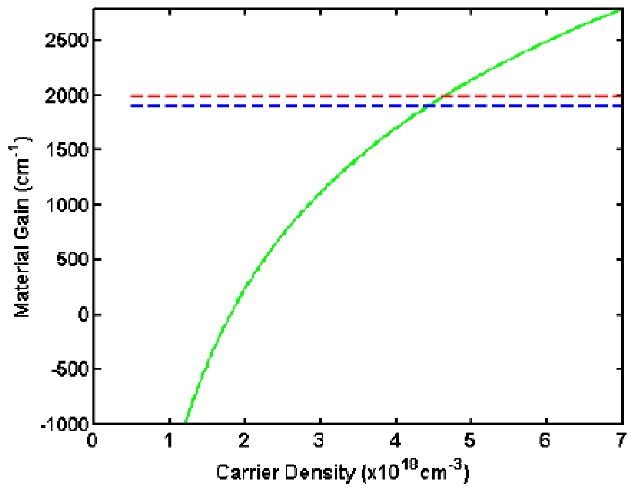

Figure 2.2: Empirical gain model for InGaAs QW, from Coldren.

(D) L-I CURVES The two curves are plotted in Fig. 4.3. The blue curve is for the laser with higher mirror reflectivity (lower loss) the red for the laser with lower reflectivity (higher loss).

Note that the additional mirror loss requires a larger threshold gain, which translates into a larger threshold current. In addition, the laser with higher mirror loss has a slightly steeper slope, though it is hard to see it on the plot.

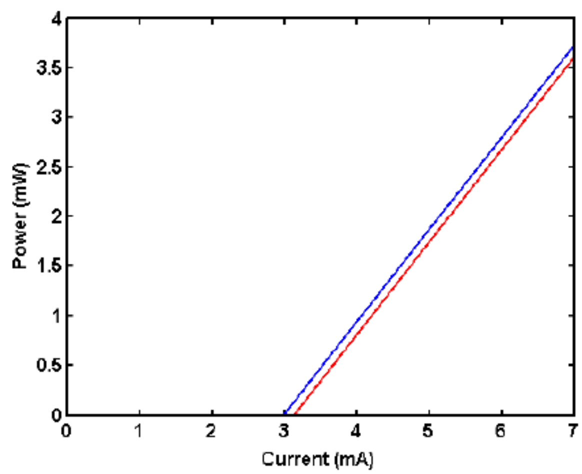

Figure 2.3: L-I curves for the two lasers. Notice the change in threshold. Also, though the difference is too small to see at this scale, there is also a difference in slope due to the difference in mirror losses.

Problem 3)

A distributed feedback laser consists of an active medium in which a periodic thickness variation is produced in one of the cladding layers forming part of the heterostructure. Owing to this structure, the mode oscillating in the laser cavity experiences a modulation of the effective refractive index along the propagation direction. This modulation can be represented by

$$n_{eff}(z) = n_0 + n_1 \sin\left[\left(2\pi z/\Lambda\right) + \varphi\right]$$

where Λ is the period of oscillation. Modulation of the refractive index induces reflections (scattering) of the laser mode in both forward and backward direction. According to Bragg's diffraction theory, constructive interference develops if the following relation holds

$$\lambda = \lambda_B = 2\langle n_{eff} \rangle \Lambda$$

where

λ = mode wavelength

λ_B = Bragg wavelength

$\langle n_{eff} \rangle$ = average value of the refractive index in the cavity

Since it is simply $\langle n_{eff} \rangle = n_0$, we can write

$$\Lambda = \frac{\lambda}{2n_0} = \frac{1.55}{2\times 3.5} = 0.2214\,\mu\text{m} = 221.4\,\text{nm}$$